fall; for which he had little or no care taken of him. He has ever fince this accident had fome complaints in his fide at times, but not conftantly; nor have they ever been fo bad, as to prevent his acting in his bufinefs as a failor, till within a few weeks before he applied to me.

London, Hatton Garden,
   June 28, 1753.

XL. *Extract of a Letter from Mr.* James Dodfon *to Mr.* William Mountaine, *F.R.S.*

May 26, 1753.

THE world has, without difpute, been obliged to the invention of fluxions, for many concife methods of calculating the peripheries, areas, and folidities, of curvilinear figures; but it muft be confeffed, at the fame time, that the moft ufeful, even of thofe, had been computed before, tho' by methods more laborious; and, confequently, fince the truth of the principles of fluxions was long difputed, that art feems rather to have received, than to have afforded, any advantage, in thofe cafes.

Neper and Briggs calculated their feveral tables of logarithms, with almoft infuperable labour; and Van Ceulen was rendered famous for his approximation to the quadrature of the circle, on account of the acknowleged tedioufnefs of its computation. The methods of computing logarithms were indeed improved, by the affiftance of the properties of the Hyperbola, and

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the ufe of infinite feries, before the great inventer of fluxions was known in the world: but then, the bufinefs of logarithms being purely arithmetical, the Hyperbola was foreign to the fubject; and the nature of infinite feries, tho' well adapted to the purpofe, was at that time but little underftood.

I am fometimes induced to believe, that, if the latter (*i. e.* the nature and ufe of infinite feries) had arrived at any degree of perfection, before the invention of fluxions, moft of the feries, which are given for the above-named kind of calculations, and have been deduced from fluxional procefles, would have been difcovered without the affiftance of them: and I am of this opinion, becaufe I am certain, that many of them might have fo been: to inftance in both the cafes above quoted; *viz.* the feries, for making logarithms, and for rectifying the circle.

And firft, the terms of one of the moft fimple feries, for expreffing the logarithm of a given number, is compofed of the powers of the excefs of that number, above unity, divided by their refpective indices; of which the firft, third, fifth, &c. terms are affirmative, and the fecond, fourth, fixth, &c. terms are negative; and the difference between the fums of the affirmative and the negative terms, is the Neperian, hyperbolic, or (as fome call it) the natural logarithm of the given number.

Now a mathematician, who underftands the nature and management of feries (altho' wholly ignorant of fluxions, or what the juftly celebrated Dr. Halley, in his admired inveftigation of this very feries, publifhed in N° 216 of the *Philofophical Tranfactions*, calls *ratiunculæ*, &c.) might arrive at the fame conclufion, in the following manner:

Since

Since the logarithm of unity is univerſally deter-
mined to be nothing; that of 2, 3, 4, 10, or any
other number, confider'd as a root, is one; that of
4, 9, 16, 100, &c. confider'd as the ſquare of that
root, is two; and ſo on; it follows, that (in all caſes)
the logarithm of a greater number will exceed that of
its leſſer; and each logarithm will have ſome rela-
tion to the exceſs of its number, above unity, the
number, whoſe logarithm is nothing : the terms of
the ſeries, therefore, which will repreſent the loga-
rithm of any number, will confiſt of the powers of
the exceſs of that number, above unity, with ſome,
yet unknown, but conſtant coefficients.

That the logarithm of the ſquare of any number
is twice the logarithm of its root, is a well-known
property of thoſe artificial numbers; and therefore,
the doubles of the particular terms of the aſſumed
ſeries will conſtitute a ſeries, expreſſing the logarithm
of the ſquare of the given number.

But, by the fourth propoſition of the ſecond book
of Euclid, the ſquare of any quantity is equal the
ſum of the ſquares of its two parts, more a double
rectangle of thoſe parts; which, in this caſe (where
the given number has been aſſumed, to confiſt of
unity, and an exceſs) will be unity, more twice that
exceſs, more the ſquare thereof.

If, therefore, the ſeveral powers of the compound
quantity (twice the exceſs of the given number above
unity, more the ſquare thereof) be multiplied by the
above aſſumed coefficients, and afterwards ranged
under each other, according to the powers of the
ſaid exceſs; their ſums will again expreſs the loga-
rithm of the ſquare of the given number.

Now,

Now, fince the logarithm of the fquare of the given number may be thus expreffed by two infinite feries, each conftituted of the excefs thereof, above unity, and its powers; it follows, that the coefficients of the like powers of that excefs, in each feries, will be equal between themfelves; and, confequently, the values of the unknown coefficients may be obtained, by fimple equations; and thefe coefficients will, by the procefs annexed, appear to be, the reciprocals of the feveral indexes of the powers of that excefs, affected alternately with the figns $+$ and $—$, as was before found, by the quadrature of the Hyperbola, by Dr. Halley in the above-cited *Philofophical Tranfaction*, and by many who have ufed a fluxional procefs.

But, there is another logarithmic feries equally fimple with the former, confifting of the fame terms, but all affirmative. This has been demonftrated to be the logarithm of that fraction, whofe numerator is unity, and denominator a number, as much lefs than unity, as the former number exceeded it.

Now, if an infinite feries be formed from that fraction, by actual divifion, it will confift of unity, and all the powers of that defect; and if the feveral powers of the excefs of this infinite feries above unity, be multiplied by the coefficients above-found, and ranged according to the powers of that defect, their fums will exhibit the above-defcribed feries for the logarithm of that fraction, as appears by the operaration fubjoin'd.

As to the application of thefe two feries, and their fum, to the finding the logarithms of numbers; the fame, being copioufly treated of by Dr. Halley (in the *Philofophical Tranfaction* before quoted) there is

no

no occasion for the repetition thereof here. *Note,* The above tract of Dr. Halley's is printed with the explication of Sherwin's tables of logarithms (fol. 10) with many examples of the ufe thereof annexed.

Secondly, The terms of one of the beft feries, for the rectification of the circle, are compofed of the odd powers of the tangent of any arc, not exceeding 45 degrees, feverally divided by their refpective indexes ; of which the firft, third, fifth, &c. terms are affirmative; and the fecond, fourth, fixth, &c. terms are negative; and the difference, between the fums of the affirmative and negative terms, is the length of that arc, of which the tangent, and its powers, conftitute the feries.

Now a mathematician, who underftands the nature and management of feries, altho' wholly ignorant of fluxions, might inveftigate this feries in the following manner :

It has been geometrically demonftrated, that, the radius of a circle being unity, if the double of the tangent of any arc, be divided by the difference between unity, and the fquare of that tangent, the quotient will be the tangent of twice the arc.

Now if an infinite feries be formed by actual divifion, its terms will confift of the doubles of the odd powers of the tangent, and will be all affirmative ; which feries will exprefs the length of the tangent of the double of that arc, whofe tangent and its powers conftitute the fame.

If a feries, confifting of the tangent and its powers, with unknown coefficients, be affumed (as in the former cafe) to exprefs the length of the arc; then the length of the double of that arc may be expreffed two ways ; *viz.* either by multiplying each term of

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the feries affumed by the number two; or by finding the powers of the feries above defcribed (which exhibits the length of the tangent of the double arc) multiplying each power by its proper coefficient, ranging the products under each other (according to the powers of the tangent of the fingle arc) and finding their fum.

Now, fince the length of the double arc may be thus expreffed, by two infinite feries, each conftituted of the tangent of the fingle arc, and its powers; therefore the coefficients of the like powers of that tangent, in each feries, will be equal between themfelves; and confequently the values of the unknown coefficients may be obtained by fimple equations.

Laftly, fince the feries, which gives the length of the tangent of the double arc confifts only of the odd powers of the tangent of the fingle arc, therefore none of the even powers thereof can range therewith: now thefe will not occur in the odd powers of that feries; and therefore the feries, affumed to exprefs the length of the fingle arc, whofe double is to be compared with the fum of the former, muft confift only of the odd powers of that tangent; and then the feries firft mentioned refults from the operation, as will appear by examining the fame, as hereto annexed.

I am thoroughly fenfible, that, to the learned, who are already mafters of the inveftigation of thefe feries by other methods, this will appear to be an effay of more curiofity than ufe; but, with regard to ftudents, I humbly conceive it will have its advantages; becaufe by thofe, who are acquainted with the notation of algebra, and the manner of folving

fimple

fimple equations, the nature of the feries, and the operaticns above ufed, may be eafily obtained; and they may be hereby enabled to make a fcientific ufe of logarithms in arithmetical, and of the rectification of the circle, in geometrical calculations; which at prefent cannot be done, till a much greater progrefs is made in mathematical knowlege, without great labour; as well as to examine and correct the printed tables of both forts, if dubious, or found to be erroneous.

The operation neceffary to find the coefficients of a feries, which will exprefs the logarithm of a given number.

If the given number be reprefented by $1+n$, then the following feries may be affumed to reprefent its logarithm:

$$n + xn^2 + yn^3 + zn^4 + un^5, \&c.$$

And $2n + 2xn^2 + 2yn^3 + 2zn^4 + 2un^5, \&c.$ will reprefent the logarithm of the fquare of that number; *viz.* of $1 + 2n + nn$.

But, becaufe $2n+nn$ is the excefs of $1 + 2n + nn$, above unity, therefore its logarithm will be alfo expreffed by

$$\overline{2n+nn} + x \times \overline{2n+nn}^2 + y \times \overline{2n+nn}^3 + z \times \overline{2n+nn}^4, \&c.$$

Now
$$\overline{2n+nn}^2 = 4nn + 4n^3 + n^4$$
$$\overline{2n+nn}^3 = 8n^3 + 12n^4 + 6n^5, \&c.$$
$$\overline{2n+nn}^4 = 16n^4 + 32n^5, \&c.$$
$$\overline{2n+nn}^5 = 32n^5, \&c.$$

Therefore,

$$\overline{2n+nn} = 2n+nn\,;$$
$$x\times\overline{2n+nn}^2 = 4xnn+4xn^3+xn^4$$
$$y\times\overline{2n+nn}^3 = 8yn^3+12yn^4+6yn^5\ \&c.$$
$$z\times\overline{2n+nn}^4 = 16zn^4+32zn^5\ \&c.$$
$$u\times\overline{2n+nn}^5 = 32un^5\ \&c.$$

And the fum of thefe is equal to the logarithm of the fquare of $1+n$.

If an equation be formed, of the coefficients of n^2, in each of thefe expreffions of the logarithm of that fquare,

Then $2x = 1 + 4x$; whence $-\frac{1}{2} = x$.

And, by proceeding in the fame manner with the coefficients of n^3, n^4, n^5, $\&c.$ and fupplying the places of x, y, z, $\&c.$ as they arife, by the numbers (fo found) we fhall have

$$2y = -\tfrac{4}{2}+8y\,;\qquad\text{whence }+\tfrac{1}{3}=y\,;$$
$$2z = -\tfrac{1}{2}+\tfrac{11}{3}+16z\,;\quad\text{whence }-\tfrac{1}{4}=z\,;$$
$$2u = \tfrac{6}{3}-\tfrac{3}{4}+32u\,;\quad\text{whence }+\tfrac{1}{5}=u\,;$$

Confequently, the logarithm of $1+n$ will be ex-preffed by $n-\frac{1}{2}n^2+\frac{1}{3}n^3-\frac{1}{4}n^4+\frac{1}{5}n^5$, $\&c.$ as above afferted.

Again, fince $\dfrac{1}{1-n}=1+n+n^2+n^3+n^4+n^5$, $\&c.$

as appears by the following divifion :

$$1-n\,)\,1\quad(\,1+n+n^2,\,\&c.$$
$$\underline{1-n}$$
$$+n$$
$$\underline{+n-nn}$$
$$+nn$$

And, fince the excefs of that feries, above unity, is the feries $n+n^2+n^3+n^4$, $\&c.$

Therefore

Therefore the logarithm of $\dfrac{1}{1-n}$ will confiſt of the ſums of the powers of that ſeries, multiplied by the above-found coefficients $\frac{1}{1}, -\frac{1}{2}, +\frac{1}{3}, -\frac{1}{4}, +\frac{1}{5}$, &c.

Now the $\begin{Bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{Bmatrix}$ Power of that Series will be $\begin{Bmatrix} n^2 + 2n^3 + 3n^4 + 4n^5, \&c. \\ n^3 + 3n^4 + 6n^5, \&c. \\ n^4 + 4n^5, \&c. \\ n^5, \&c. \end{Bmatrix}$

And,

$\begin{Bmatrix} \frac{1}{1} \\ -\frac{1}{2} \\ +\frac{1}{3} \\ -\frac{1}{4} \\ +\frac{1}{5} \end{Bmatrix}$ of that $\begin{Bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{Bmatrix}$ Power will be $\begin{Bmatrix} n + n^2 + n^3 + n^4 + n^5, \&c. \\ -\frac{1}{2}n^2 - \frac{2}{2}n^3 - \frac{3}{2}n^4 - \frac{4}{2}n^5, \&c. \\ +\frac{1}{3}n^3 + \frac{3}{3}n^4 + \frac{6}{3}n^5, \&c. \\ -\frac{1}{4}n^4 - \frac{4}{4}n^5, \&c. \\ +\frac{1}{5}n^5, \&c. \end{Bmatrix}$

The ſums of which, *viz.* $\overline{n + \frac{1}{2}n^2 + \frac{1}{3}n^3 + \frac{1}{4}n^4 + \frac{1}{5}n^5}$, &c. will be the logarithm of $\dfrac{1}{1-n}$, as above affirmed.

The operation neceſſary, to find the coefficients of a ſeries, which will expreſs the length of the arc of a circle, by the tangent of that arc, and its powers.

Let *a* repreſent the length of the arc, and *t* the length of the tangent thereof; then the tangent of that arc whoſe length is 2*a*, will be $\dfrac{2t}{1-tt}$; which fraction is equal to the infinite ſeries, $2t + 2t^3 + 2t^5 + 2t^7 + 2t^9$, &c. See the diviſion.

N n

$1 - tt$

$$1-tt \,)2t \quad \quad (\, 2t + 2t^3 + 2t^5, \&c.$$
$$\quad \quad 2t - 2t^3$$
$$\quad \quad \overline{\quad + 2t^3}$$
$$\quad \quad \quad + 2t^3 - 2t^5$$
$$\quad \quad \quad \overline{\quad \quad + 2t^5}$$

And by performing the neceſſary multiplications, or diviſions, it will alſo appear, that

$$\overline{\frac{2t}{1-tt}}^3 = \quad 8t^3 + 24t^5 + \quad 48t^7 + \quad 80t^9, \&c.$$

$$\overline{\frac{2t}{1-tt}}^5 = \quad \quad \quad 32t^5 + 160t^7 + 480t^9, \&c.$$

$$\overline{\frac{2t}{1-tt}}^7 = \quad \quad \quad \quad 128t^7 + 896t^9, \&c.$$

$$\overline{\frac{2t}{1-tt}}^9 = \quad \quad \quad \quad \quad \quad 512t^9, \&c.$$

Now if we aſſume, for the value of a, the follow-ing ſeries, $t + xt^3 + yt^5 + zt^7 + ut^9$, &c.

Then $2t + 2xt^3 + 2yt^5 + 2zt^7 + 2ut^9$, &c. $= 2a$.

And becauſe $\frac{2t}{1-tt}$ is the tangent of the arc, whoſe length is $2a$, therefore

$$\frac{2t}{1-tt} + x \times \overline{\frac{2t}{1-tt}}^3 + y \times \overline{\frac{2t}{1-tt}}^5 + z \times \overline{\frac{2t}{1-tt}}^7 \&c. = 2a.$$

Which expreſſion is equivalent to the ſum of the following ſeries ; for

$$\frac{2t}{1-tt} = 2t + 2t^3 + 2t^5 + 2t^7 + 2t^9, \ \&c.$$

$$x \times \overline{\frac{2t}{1-tt}}\Big|^3 = 8xt^3 + 24xt^5 + 48xt^7 + 80xt^9, \ \&c.$$

$$y \times \overline{\frac{2t}{1-tt}}\Big| = 32yt^5 + 160yt^7 + 480yt^9, \ \&c.$$

$$z \times \overline{\frac{2t}{1-tt}}\Big| = 128zt^7 + 896zt^9, \ \&c.$$

$$u \times \overline{\frac{2t}{1-tt}}\Big| = 512ut^9, \ \&c.$$

And (by making an equation, between $2x$, the coefficient of t^3 in the firſt-found value of $2a$, and $2+8x$, the ſum of the coefficients of t^3 in the latter) $2x = 2 + 8x$; whence $-\frac{1}{3} = x$.

And, by proceding in the ſame manner, with the coefficients of t^5, t^7, t^9, $\&c.$ and ſupplying the places of x, y, z, &c. (as they ariſe) by the numbers (ſo found) we ſhall have

$$2y = 2 - \frac{24}{3} + 32y; \qquad \text{whence } y = +\tfrac{1}{5}.$$
$$2z = 2 - \frac{48}{5} + \frac{160}{5} + 128z; \qquad \text{whence } z = -\tfrac{1}{7}.$$
And $2u = 2 - \frac{80}{3} + \frac{480}{5} - \frac{896}{7} + 512u$; whence $u = +\tfrac{1}{9}$.

Therefore we may conclude, that $t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7 + \frac{1}{9}t^9, \&c. = a$.

The application of this ſeries to the rectification of the circle is extant in many authors; particularly in Sherwin's tables of logarithms, above-quoted, folio 55.

When the arc is juſt 45 degrees, then $t = 1$, and the ſeries becomes $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}, \&c.$ which converges exceedingly ſlow; but, by the aſſiſtance of a

method,

method, given in the appendix to M. De Moivre's *Miſcellanea Analytica*, it may be transformed to another, converging quicker ; which method is applied to this very ſeries, in folio 362 of the *Mathematical Repoſitory*, Vol. I.

XLI. *A Letter from* John Lining, *M. D. of* Charles-Town, South-Carolina, *to the Rev.* Thomas Birch, *D. D. Secr. R. S. concerning the Quantity of Rain fallen there from* January 1738, *to* December 1752.

Rev. Sir,　South Carolina, Charles-Town, April 9, 1753.

Read July 8, 1753. THE favourable reception, which my former papers met with from the Royal Society, encourages me to ſend you a table of the quantity of rain, which fell in Charles-Town for theſe 15 years laſt paſt ; which, if continued for half a century, might be of uſe, in diſcovering to us the changes made in a climate, by clearing the land of its woods. Tho' I formerly ſent a table of the rain from 1738 to 1745 incluſive, which is publiſh'd in N° 487 of the *Philoſophical Tranſactions* ; yet, as I thought it would be more convenient to bring the whole into one view, I have not only added to this table the rain of thoſe years, but have likewiſe diſtinguiſhed the quantity which fell in the ſeveral ſeaſons. In this table I continued the old ſtile to the firſt of laſt February, that the mean quantity, in each month, and in the different ſeaſons, might be given exactly.

As